ENERGY CHARACTERISTICS OF HARMONIC

INTERNAL WAVE GENERATORS

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The propagation of internal waves plays an important role in liquid media with layers that vary according to density (stratified liquids) and are located in a gravitational field, which include the Earth's atmosphere and oceans. Highly controlled experiments are essential for investigating efficient generators of internal waves (in particular, harmonic internal waves). Hence, it is important to compare the efficiences of various types of internal wave generators. This problem is considered for the simplest forms of stratification: discontinuous and uniform (with a constant buoyancy frequency N). Although there are very few studies of oscillations in the case of discontinuous stratification, there are even fewer investigations of uniform stratification (e.g., see [1-4]). A comparison of the efficiences of different types of generators has not been made for the latter case. This is done below on the basis of energy estimates for two types of generators: for objects (a sphere or cylinder) that undergo small harmonic oscillations in a liquid and for objects with pulsating volumes.

1. A Two-Layer Liquid. We will consider two uniform, incompressible, ideal liquids of different densities that extend without limit above and below a dividing surface z = 0. The results are based on a direct generalization of the data in [5] for a single liquid with a free surface but have been obtained by other methods.

We will model the oscillating objects with a distribution of massive sources $m(r, t) = m_0(r) \sin \omega_0 t$. Then, small irrotational perturbations of the velocity potential in the two-layer liquid $\phi(r, t)$ satisfy the equations

$$\Delta\varphi = m, \left[\frac{\partial\varphi}{\partial z}\right] = 0, \left[\rho\left(\frac{\partial^2\varphi}{\partial t^2} - g\frac{\partial\varphi}{\partial z}\right)\right] = 0, \qquad (1.1)$$
$$\mathbf{v} = \nabla\varphi, \ p = -\rho\frac{\partial\varphi}{\partial t}, \ p, \mathbf{v} \underset{r\to\infty}{\longrightarrow} 0.$$

Here t, z are the time and the vertical coordinate; v, p are the velocity and pressure perturbations; r = {x, y, z}, r = {x, z} are the three-dimensional and two-dimensional problems, respectively (also, r_h = {x, y, 0} and r_h = {x, 0}); $[f] \equiv f|_{z=+0} - f|_{z=-0}$.

The potential can be put into the form of an integral convolution of the massive source with a Green retardation function $G^{ret}(r_h, z, z', t)$, which is the solution of system (1.1) with an instantaneous point source $m(r, t) = \delta(t)\delta(r_h)\delta(z - z')$ and with the causality condition $G^{ret} = (r_h, z, z', t)|_{t<0} = 0$ [6]:

$$\varphi(\mathbf{r}_h, z, t) = \int dz' d^{p-1}\mathbf{r}'_h dt' G^{ret}(\mathbf{r}_h - \mathbf{r}'_h, z, z', t - t') m(\mathbf{r}'_h, z', t').$$

After substituting this representation into the equation for the average (over one period of the oscillation) power dissipated upon the formation of internal waves at the surface discontinuity of the density

$$\langle W\rangle = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} dt \int dz d^{p-1}\mathbf{r}_h p(\mathbf{r}_h, z, t)\, m(\mathbf{r}_h, z, t) \qquad (1.2)$$

one can carry out simple calculations to show that only the imaginary part of the Fourier form of the Green retardation function makes a contribution to <W> (after making the

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Fourier transformation, we will use the same definitions as those for the initial quantities, except that $r \leftrightarrow k$, $t \leftrightarrow \omega$):

$$\langle W\rangle = -\frac{\rho\omega_0}{2(2\pi)^{p-1}}\int dz dz' d^{p-1}\mathbf{k}_h m_0(\mathbf{k}_h, z')\, m_0(-\mathbf{k}_h, z)\times \times \operatorname{Im} G^{\text{ret}}(\mathbf{k}_h, z, z', \omega_0). \tag{1.3}$$

The existence of only the imaginary part of the Green function simplifies calculations since it is proportional to the δ-function, which is concentrated on the surface assigned by the dispersion equation for free waves. We have the following for two unbounded layers of uniform liquids

$$\operatorname{Im} G^{\text{ret}}(\mathbf{k}_h, z, z', \omega) = -\frac{\pi\omega z}{2|\omega z|}\left(\gamma + \frac{z'}{|z'|}\right) e^{-k_h|z| - k_h|z'|}\,\delta\left(k_h - \frac{\omega^2}{\gamma g}\right),$$

and the equation for the power loss takes the form

$$\langle W\rangle = \frac{\omega_0\rho(1\pm\gamma)}{8(2\pi)^{p-2}}\int d^{p-1}\mathbf{k}_h\delta\left(k_h - \frac{\omega_0^2}{\gamma g}\right)\left|\int dz e^{-k_h|z|} m_0(\mathbf{k}_h, z)\right|^2. \tag{1.4}$$

Here, the sign +(−) and $\rho = \rho_2 (=\rho_1)$ correspond to the case when the massive source is below (above) the boundary interface; $\gamma \equiv (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$; and p is the dimensionality of the space.

For obtaining the results for real objects, it is necessary to model the objects by systems of massive sources. However, for a complete consideration of the approximations, we will use models from the theory of uniform unbounded liquids. One can hope for satisfactory results only for objects that are distant from the boundary interface. This approach is used, in particular, when considering waves on a free surface in [5], where the solutions of the bounded integral equations are applied.

In a uniform, unbounded, ideal, incompressible liquid, the flow outside the cylinder (p = 2) or a sphere (p = 3), whose volumes change in a weakly harmonic manner, coincides with the flow from a massive source

$$m(\mathbf{r}, t) = -2\pi a\omega_0 r_0 (2r_0)^{p-2}\delta(\mathbf{r}_h)\delta(z - z_0)\sin\omega_0 t. \tag{1.5}$$

Using the same source for a two-layer liquid, we find from (1.4) that the energy dissipated in a single unit of time by a pulsating cylinder or sphere (with a radius of $r_0 + a\cos\omega_0 t$) due to the formation of waves on the surface discontinuity of the density is given by the equation

$$\langle W^{(1)}\rangle = \pi^2 a^2 r_0^2\omega_0^3\rho(1\pm\gamma)\left(\frac{2r_0^2\omega_0^2}{\gamma g}\right)^{p-2}\exp\left(-\frac{2\omega_0^2|z_0|}{\gamma g}\right), \tag{1.6}$$

which in the limit as $\gamma \to 1$ becomes (4.13), (6.8) from [5].

In an unbounded, uniform, ideal, incompressible liquid, there arises a flow outside the cylinders and spheres undergoing small oscillations in the direction of the vector a that coincides with the flow from a dipole massive source

$$m(\mathbf{r}, t) = 2\pi r_0^p\omega_0\mathbf{a}\nabla\delta(\mathbf{r}_h)\,\delta(z - z_0)\sin\omega_0 t. \tag{1.7}$$

An estimate made with this source for the energy loss from the oscillating objects to the emission of internal waves on the boundary interface has the following form according to (1.4)

$$\langle W^{(2)}\rangle = c_p\frac{\pi^2}{4}\rho(1\pm\gamma)\omega_0^3\left(\frac{2r_0^2\omega_0^2}{\gamma g}\right)^p\exp\left(-\frac{2\omega_0^2|z_0|}{\gamma g}\right), \tag{1.8}$$

$$c_2 = a^2 = a_x^2 + a_z^2,\quad 4c_3 = a_z^2 + \frac{1}{2}(a_x^2 + a_y^2).$$

As $\gamma \to 1$, we obtain the corresponding equations from [5].

Equations (1.6) and (1.8) allow one to compare the efficiencies of two types (pulsating and oscillating) of internal wave generators. The ratio of the powers

$$\langle \dot{W}^{(2)}\rangle/\langle W^{(1)}\rangle = \frac{c_p}{a^2}\left(\frac{r_0\omega_0^2}{\gamma g}\right)^2$$

is proportional to the square of the ratio of the object's dimension to the characteristic wavelength $r_0/(\gamma g\omega_0^{-2})$, since for small oscillation frequencies ($\omega_0^2 < \gamma g/r_0$), volumetric vibrators are more efficient (this is valid up to $r_0\omega_0^2/\gamma g \approx 1$ because of the smallness of the factor c_p/a^2). For sufficiently high frequencies ($\omega_0^2 \gg \gamma g/r_0$), one can expect the reverse situation. However, the characteristic wavelength ($\sim\gamma g/\omega_0^2$) is much less than the size of the object (r_0), and the use of point sources for modeling is doubtful because of the importance of interference effects at scales of $\sim r_0$.

2. A Uniformly Stratified, Unbounded Liquid. Small perturbations in the pressure p due to a massive source $m(r, t) = m_0(r)\sin\omega_0 t$ in an initially motionless, incompressible, ideal, stratified liquid with a constant buoyancy frequency N in the Boussinesq approximation (we will use a system of units in which $\rho_0 = 1$) satisfy the equation [6]

$$\hat{L}p = -\omega_0(N^2-\omega_0^2)m_0(\mathbf{r})\cos\omega_0 t,\ \hat{L} \equiv \frac{\partial^2}{\partial t^2}\nabla^2 + N^2\nabla_h^2. \tag{2.1}$$

Solving this equation using the Green retardation function for the operator $\hat{L}$, one can find an equation of the type (1.3) for the average power losses (1.2)

$$\langle W\rangle = -\frac{\omega_0(N^2-\omega_0^2)}{2(2\pi)^p}\int d^p\mathbf{k}\,|m_0(\mathbf{k})|^2\,\mathrm{Im}\,G^{\mathrm{ret}}(\mathbf{k},\omega_0).$$

Since the imaginary part of the Fourier form of the Green function is proportional to the δ-function

$$\mathrm{Im}\,G^{\mathrm{ret}}(\mathbf{k},\omega) = -\pi\frac{\omega}{|\omega|}\delta(\omega^2k^2 - N^2k_h^2),$$

in the equation for the power loss to emission of internal waves

$$\langle W\rangle = \frac{\omega_0(N^2-\omega_0^2)}{4(2\pi)^{p-1}}\int d^p\mathbf{k}\,|m_0(\mathbf{k})|^2\,\delta(\omega_0^2k^2 - N^2k_h^2) \tag{2.2}$$

one of the integrations can be done in the general case.

For symmetric (spherically or cylindrically) sources, which include point-like monopole sources (1.5), one can make a simplification that is related to integration over all angles. The equation for the losses takes the form ($\omega_0 \le N$)

$$\langle W\rangle = \left(\frac{2r_0^2\omega_0}{N}\right)^p\frac{\pi N^2a^2\sqrt{N^2-\omega_0^2}}{4r_0^2}\int\limits_0^\infty dk k^{p-3}\mu^2(k),$$

and it follows that it is impossible to estimate the energy losses by modeling cylinders and spheres pulsating and oscillating in an uniform, stratified liquid with point sources (1.5), (1.7). Actually, a point-like monopole source (1.5) corresponds to $\mu^2(k) = 1$, and the integral over the wave numbers in the equation for the power loss diverges (logarithmically for large and small wave numbers in the two-dimensional problem and linearly for large k in the three-dimensional problem). For a point-like dipole source (1.7), we have $\mu^2 \sim k^2$ and the divergence of the integral for large k is even greater (compare with the paradox of infinite energy losses in the problem of uniform motion of point sources [6, 7]). To avoid this difficulty, one must model the oscillating objects with nonlocalized sources. Hence, we will use distribution surfaces for the massive sources.

In a uniform, ideal, incompressible liquid, the flow around the sphere (cylinder) for small harmonic changes in the volume coincides with the flow due to two-dimensional massive sources

$$m(\mathbf{r},t) = m_0(\mathbf{r})\sin\omega_0 t,\ m_0(\mathbf{r}) = -\omega_0 a\delta(r-r_0). \tag{2.3}$$

Using this distribution to model a sphere pulsating in a uniformly stratified liquid, we obtain the following simple result for the average power lost to the formation of internal waves

$$\langle W\rangle = \pi^2 a^2 r_0^3 N^{-1}\omega_0^3 \sqrt{N^2-\omega_0^2}, \quad \omega_0 \leqslant N. \tag{2.4}$$

According to this equation, maximal losses occur for $\omega_0 = N\sqrt{3/2}$.

In the two-dimensional problem of a pulsating cylinder modeled by a distributed massive source (2.3), the form-factor $\mu^2(k)$ is proportional to $J_0^2(kr_0)$, which guarantees the convergence of the integral for large wave numbers, but the logarithmic divergence for small wave numbers remains. This divergence can be eliminated by assuming that the dimensions of the basin are restricted (see section 3). One obtains positive results by avoiding the Boussinesq approximation. However, this is related to changes in the scales $\gtrsim g/N^2$, which usually greatly exceed the dimensions of the basin.

Borrowing the dipole distribution from the theory of a uniform liquid

$$m(\mathbf{r}, t) = -\left(\frac{3}{4}\right)^{p-2} 2\omega_0 \frac{\mathbf{ra}}{|\mathbf{r}|}\delta(r-r_0)\sin\omega_0 t \tag{2.5}$$

we obtain the following for the average power loss due to the formation of internal waves for the model of a sphere (p = 3) and a cylinder (p = 2) of radius r_0 from (2.3) oscillating in a stratified liquid along the direction of vector **a**

$$\langle W^{(2)}\rangle = 2\pi r_0^2\omega_0^2\sqrt{N^2-\omega_0^2}\left(\frac{3\pi\omega_0 r_0}{8N}\right)^{p-2} A_p, \tag{2.6}$$

$$A_2 \equiv a_x^2\frac{\omega_0^2}{N^2} + a_z^2\left(1-\frac{\omega_0^2}{N^2}\right), \quad A_3 \equiv (a_x^2+a_y^2)\frac{\omega_0^2}{2N^2} + a_z^2\left(1-\frac{\omega_0^2}{N^2}\right).$$

Comparing expressions (2.4) and (2.6)

$$\langle W^{(2)}\rangle/\langle W^{(1)}\rangle = \frac{3}{4}\left\{\frac{\omega_0^2}{2N^2}\sin^2\theta + \left(1-\frac{\omega_0^2}{N^2}\right)\cos^2\theta\right\},$$
$$\mathbf{a} = a\{\cos\varphi\sin\theta,\ \sin\varphi\sin\theta,\ \cos\theta\},$$

one can conclude that, with the exception of very small frequencies for a horizontally oscillating object and $\omega_0 \lesssim N$ for vertically oscillating objects, pulsating and oscillating spheres are characterized by the same efficiency in relation to the creation of internal waves in an unbounded, uniformly stratified liquid.

This conclusion is made based on modeling by the sources (2.3) and (2.5). One can only hope that it is satisfactory for an oscillation in the volume of the liquid that is small in comparison with the wave volume, i.e., for the condition $a \ll r_0$.

3. A Stratified Liquid in a Horizontal Waveguide. We will consider a stratified liquid with $N = N(z)$ between two rigid horizontal planes $z = 0$ and $z = H$ that is weakly perturbed by a periodic massive source $m(r, t) = m_0(r)\sin\omega_0 t$. We have the following for the perturbations of the vertical component of the velocity w [6]

$$\hat{L}w = -\omega_0^2\frac{\partial m_0}{\partial z}\sin\omega_0 t, \quad w|_{z=0} = w|_{z=H} = 0,$$

and the Fourier component of the pressure perturbation is

$$p(\mathbf{k}_h, z, \omega) = -\frac{i\omega}{k_h^2}\left\{m(\mathbf{k}_h, z, \omega) - \frac{\partial w(\mathbf{k}_h, z, \omega)}{\partial z}\right\}.$$

As earlier, using the Fourier transformation in terms of the horizontal coordinates and time and applying an expansion over the vertical coordinate in terms of the eigenfunctions of the problem

$$\left\{\frac{\partial^2}{\partial z^2} - k_h^2 + \frac{1}{\omega_n^2}k_h^2 N^2(z)\right\}\psi_n(k_h, z) = 0,$$
$$\psi_n|_{z=0} = \psi_n|_{z=H} = 0,$$

one can represent the average power loss to radiation of internal waves (1.2) in the form of the following expression, which is squared in terms of the massive source

$$\langle W\rangle = -\frac{\omega_0^3}{2(2\pi)^{p-1}}\int d_{\cdot}^{p-1}k_h\int_0^H dz dz' m_0(\mathbf{k}_h, z')\, m_0(\mathbf{k}_h, z)\times$$
$$\times\frac{1}{k_h^2}\frac{\partial^2 \operatorname{Im} G^{\mathrm{ret}}(\mathbf{k}_h, z, z', \omega_0)}{\partial z\partial z'},$$

where only the imaginary part of the Fourier form of the Green function enters. The latter is the sum of the δ-functions over all the internal wave modes

$$\operatorname{Im} G^{\mathrm{ret}}(\mathbf{k}_h, z, z', \omega) = -\pi\frac{\omega}{|\omega|}\sum_n \psi_n(k_h, z)\,\psi_n(k_h, z')\,\delta\left(\frac{\omega^2}{\omega_n^2}-1\right)$$

which allows one to put the equation for the power loss in the form

$$\langle W\rangle = \frac{\omega_0^3}{4(2\pi)^{p-2}}\int d^{p-1}\mathbf{k}_h\frac{1}{k_h^2}\sum_n|\mu_n(\mathbf{k}_h)|^2\,\delta\left(\frac{\omega_0^2}{\omega_n^2}-1\right), \tag{3.1}$$
$$\mu_n(\mathbf{k}_h)\equiv\int_0^H dz\frac{\partial\psi_n(k_h, z)}{\partial z}\,m_0(\mathbf{k}_h, z).$$

We will limit our consideration to a waveguide of thickness H filled with a uniformly stratified liquid N = const, where

$$\psi_n(k_h, z) = \frac{1}{Nk_h}\sqrt{\frac{2}{H}}\sin\frac{\pi n z}{H},\quad \omega_n^2 = \frac{N^2k_h^2}{k_h^2+\frac{\pi^2 n^2}{H^2}},\quad n = 1, 2, \ldots$$

Then, for modeling pulsating and oscillating cylinders located far from the boundary ($r_0 \ll z_0$, $r_0 \ll H - z_0$), one can use the surface distributions of the sources (2.3) and (2.5) to find from (3.1) that

$$\langle W^{(1)}\rangle = 2\pi\omega_0^2 a^2 r_0^2\sqrt{N^2-\omega_0^2}\sum_{n=1}^{\infty}\frac{\cos^2 nZ}{n}J_0^2(nR), \tag{3.2}$$
$$\langle W^{(2)}\rangle = 8\pi\omega_0^2 r_0^2\sqrt{N^2-\omega_0^2}\sum_{n=1}^{\infty}\frac{1}{n}\left\{a_x^2\frac{\omega_0^2}{N^2}\cos^2 nZ + a_z^2\left(1-\frac{\omega_0^2}{N^2}\right)\sin^2 nZ\right\}J_1^2(nR),$$
$$Z\equiv\frac{\pi z_0}{H},\quad R\equiv\pi\frac{r_0}{H}\frac{N}{\sqrt{N^2-\omega_0^2}}.$$

One should note that when making the transition in $\langle W^{(1)}\rangle$ to the limit of an unbounded medium ($z_0 \to \infty$, $H/z_0 \to \infty$), the sum over all modes logarithmically diverges, which is in full agreement with the results of section 2. The same same results are obtained by substituting the point source (1.5) into (3.1) without going to the limit $H \to \infty$.

For spheres that pulsate and oscillate in a layer of uniformly stratified liquid far from the boundaries and that are modeled by a surface distribution of massive sources (2.3), (2.5), one finds the following from (3.1)

$$\langle W^{(1)}\rangle = 4\pi r_0^3\omega_0^3 a^2 N^{-1}\sqrt{N^2-\omega_0^2}\,R^{-1}S_1, \tag{3.3}$$

$$\langle W^{(2)}\rangle = 9r_0^2\omega_0^3 HN^{-2}(N^2-\omega_0^2)\left\{(a_x^2+a_y^2)\frac{\omega_0^2}{2N^2}S_2 + a_z^2\left(1-\frac{\omega_0^2}{N^2}\right)S_3\right\},$$

$$S_1 = S_1(Z, R) \equiv \sum_{n=1}^{\infty}\frac{\cos^2 nZ\sin^2 nR}{n^2},$$

$$S_2 = S_2(Z, R) \equiv \sum_{n=1}^{\infty}\frac{1}{n^2}\cos^2 nZ\left(\cos nR - \frac{\sin nR}{nR}\right)^2,$$

$$S_3 = S_3(Z, R) \equiv \sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2 nZ\left(\cos nR - \frac{\sin nR}{nR}\right)^2.$$

It is evident from Eqs. (3.2) and (3.3) that, taking into account the conditions when $r_0 \ll H$ for oscillation frequencies that are not too close to the buoyancy frequency $\sqrt{N^2-\omega_0^2} \sim N$), the source with the pulsating volume is more effective for the excitation of lower internal wave modes. The contributions of the lower modes in the two-dimensional (3.2) and three-dimensional (3.3) cases are such that the ratios for the oscillating and pulsating objects are proportional to the square of the small ratio of the dimensions of the objects to the thickness of the waveguide $(r_0/H)^2$. One should note that the result of (1.8) when there is only a single surface mode permits an analogous interpretation, since the length $\gamma g/\omega_0^2$ characterizes the wavelength and the depth of the layer affected by the wave motion.

The Fourier series with a period of π given in terms of Z enter into (3.2) and (3.3). However, only the region $Z < \pi$ has a meaning in the above problems. Moreover, since these series are invariant relative to the substitution $Z \to \pi - Z$, it is sufficient to analyze $Z < \pi/2$ assuming that the center of the sphere or the axis of the cylinder is located in the upper half of the waveguide. The series $S_1(Z, R)$ is given in terms of the second argument R of the periodic functions, and the series S_2, S_3 are the sums of the three terms proportional to the series that are periodic in terms of both arguments Z and R.

The series are easy to sum in the three-dimensional problem with pulsating and oscillating sphere

$$S_1 = \frac{1}{2}R(\pi - R) - \frac{\pi}{8}(Z + R - |R-Z|), \quad 0 \leqslant Z,\ R \leqslant \pi/2,$$

$$S_2 = \frac{\pi}{12}\begin{cases} R, & 0 \leqslant R \leqslant Z \leqslant \pi/2, \\ 2R - 3Z + 2Z^3/R^2, & 0 \leqslant Z \leqslant R \leqslant \pi/2, \end{cases}$$

$$S_3 = \frac{\pi}{12}\begin{cases} R, & 0 \leqslant R \leqslant Z \leqslant \pi/2, \\ 3Z - 2Z^3/R^2, & 0 \leqslant Z \leqslant R \leqslant \pi/2. \end{cases}$$

The inequality $R \leq Z$ is equivalent to $\omega_0 \leq N\sqrt{1 - r_0^2/z_0^2}$, and, since $r_0 \ll z_0$ for the output condition, it is violated only if the oscillation frequency is very close to the buoyancy frequency N. On the other hand, although interesting interference oscillations of the power occur in the latter situation with an increase in the frequency ω_0, the quantities for the power become very small according to (3.3) because the additional factors are small $\sim\sqrt{N^2 - \omega_0^2}$.

Therefore, we have the following for the power loss to emission of internal waves due a pulsating sphere

$$\langle W^{(1)}\rangle = \pi^2\omega_0^3 r_0^3 a^2\left(\frac{\sqrt{N^2-\omega_0^2}}{N} - \frac{2r_0}{H}\right), \quad 0 \leqslant \omega_0 \leqslant N\sqrt{1 - r_0^2/z_0^2}.$$

It is evident from here that the simplest result (2.4) is obtained when making the transition to the limit of an unbounded liquid, and few waves are generated shortly before the limiting situation. For an oscillating sphere, the summation of the series under the same conditions when $\omega_0 \leq N\sqrt{1 - r_0^2/z_0^2}$ yields Eq. (2.6).

Hence, the total efficiencies of pulsating and oscillating generators of internal waves in a horizontal waveguide, as is true in the limit of an unbounded liquid, are equivalent. However, there is a great difference in the distributions of energy over the modes. Maximal power loss occurs at the highest modes for oscillating sources.

One should note that surface waves will be generated if the upper boundary of the waveguide is free. In relation to internal waves, there is little difference from the case considered above for the "solid cap", where r_0, $z_0 << H$.

Modeling of oscillating objects with simple distributions of massive sources is no longer satisfactory if the amplitude of the oscillations is increased. However, in the opposite extreme case with very large amplitudes ($a >> r_0$), a different type of simple modeling is possible, and the results remain fairly straightforward [8].

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